The Effective Mass of He^3 and the Velocity of Second Sound in Solutions of He^3 in He^4

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THE EFFECTIVE MASS OF He^3 AND THE VELOCITY OF SECOND SOUND IN SOLUTIONS OF He^3 IN He^4

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In solutions of He^3 in He^4, just as in pure He^4, below the λ-point, undamped thermal waves (second sound), besides ordinary sound, can be propagated. I. Ya. Pomeranchuk [1] derived a formula determining the velocity of second sound c_2 in weak solutions of He^3 in He^4:

$$c_2^2 = \frac{\rho_s}{\rho_n} \frac{(S_0 - kx/m_3)^2 T}{(C_0 + 3kx/2m_3)} + \frac{kTx}{m_3} \tag{1}$$

Here ρ_n is the density of the normal part of the solution; $\rho_s = \rho - \rho_n$ is the density of the superfluid part of the solution; S_0 is the entropy and C_0 the heat capacity of pure helium-II; $x = \frac{(S_0 N_3 m_3}{N_4 m_4 + N_3 m_3}$ is the concentration; N_3 and N_4 designate the number of atoms of He^3 and He^4, respectively, in the solution; m_3 and m_4 are the respective masses of He^3 and He^4 atoms, and T is the temperature. (NOTE: The behavior of concentrated solutions of He^3 in He^4 is also a problem under consideration. The corresponding expressions for the velocity of first and second sound in such solutions have already been obtained).

Formula (1) was derived on the assumption that the particles of the admixtures are subject to classical statistics. For sufficiently low temperatures the distribution of particles of the mixture will deviate from the classical. This phenomenon, however, occurs at very low temperatures of the order of 0.1 - 0.2°K for a solution of He^3 in He^4 with a concentration of the order 0.05.

The density ρ_n of the normal part of the liquid consists of two parts: the normal density ρ_{n4} of pure helium-II and the normal density of the mixture ρ_{n3}. The normal density of the mixture essentially depends on the energy-spectrum type of the admixtures in the solution. If zero momentum corresponds to minimum energy, then ρ_{n3} has the form

$$\rho_{n3} = \frac{\rho}{m_3} x \mu$$

where μ is the effective mass of the mixture.

In the second logically possible case, where a momentum P_o not equal to zero corresponds to energy minimum, we have according to [1]

$$\rho_{n3} = \frac{\rho}{m_3} x \frac{P_o^2}{3kT}$$

Hence in this case, in contrast to the previous, the effective mass $\mu_{ef} = p_o^2/3kT$ is found to depdnd on temperature.

Recently the velocity of second sound in solutions of He^3 in He^4 were measured, thus enabling us to make definite conclusions as to the energy spectrum of He^3 in solution [2].

In the work referred to, the authors measured the velocity of second sound in solutions of He^3 in He^4 with molar concentrations within limits of 0.09 to 0.8% and a temperature range of 1.25 to 1.7°K. (NOTE: The molar concentration $c = \frac{N_3}{N_4 + N_3}$

is related to the concentration $x = \frac{N_3 m_3}{N_4 m_4 + N_3 m_3}$ appearing in our formulas by the relation $\frac{1}{\varepsilon} - 1 = (\frac{1}{x} - 1)\frac{m_3}{m_4}$. For small concentrations we have $x = \varepsilon\frac{m_3}{m_4}$.)

The values S_o, C_o and ρ_{n4} appearing in formula (1) are known functions of temperature (see e.g., [3]. Therefore the results [2] with the help of formula (1) allow us to compute the value of the normal density ρ_{n3} of the mixture and consequently also the effective mass of He^3 in solution. Such computations gave as the effective mass the value

$$\mu = 8.5\ m_1 \qquad (m_1 = \text{proton mass})$$

The μ-values obtained from values of c_2 for various concentrations and temperatures did not diverge more than 5% of the deduced value. Therefore the effective mass was found, within the limits of accuracy of the experiment, to be independent of temperature. This result allows us to make the definite conclusion that He^3 dissolved in helium-II possesses an energy spectrum that has a minimum for the zero value of momentum; or, in other words, the energy of the admixture in solution has the form:

$$E = \frac{p^2}{2} \qquad (p:\ \text{momentum}) \qquad (4)$$

We should remark that, if the zero momentum p_o did not correspond to the minimum, then the effective mass under the conditions of the experiment [2] would vary 35% during a temperature variation from 1.25 to 1.7°K.

The authors of work [2] compared the results of their measurements with the computations by I. Ya. Pomeranchuk [1] and revealed a divergence which led them to the conclusion that a non-zero momentum p_o is present in He^3 dissolved in helium-II.

Such a conclusion was erroneous, because the computations in [1] were performed on the assumption that the effective mass of He^3 equals the atomic mass of He^3. Actually the real effective mass exceeds by nearly three times the atomic mass of He^3. The magnitude of the effective mass of He^3 enables us to anticipate the thermal behavior of the velocity of second sound in solutions of He^3 in He^4 for various concentrations. The results of computations for a temperature range of 0.6 to 1.8°K are represented in figures 1 and 2 [see the Appendix].

Submitted 23 April 1951

Bibliography

1. Pomeranchuk, I.; Zhurnal Teoreticheskoy Fiziki, Vol 19, p 42 (1949).
2. Lynton, E., Fairbank, N.; Physical Review, 80, 1043 (1950).
3. Landau, L.; Journal of Physics, Vol 11, 91 (1947).

APPENDIX

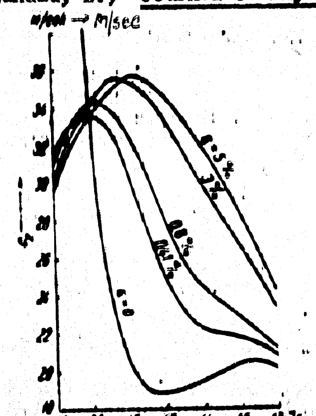

Fig 1. Temperature dependence of the velocity of second sound in solutions of He^3 in He^4 for various concentrations . The small circles [sic] represent the experimental values of the velocity [2]; the continuous curves represent the theoretical values ~~xxxxxxxxxx~~ [sic].

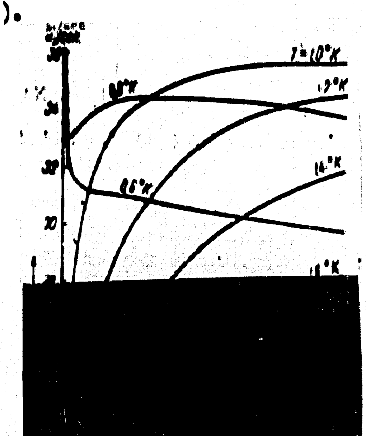

Fig 2. Dependence of the velocity of second sound in solutions of He^3 in He^4 on the concentration of the solution. The continuous curves [sic] represent the theoretical values of velocity (at an assigned temperature).

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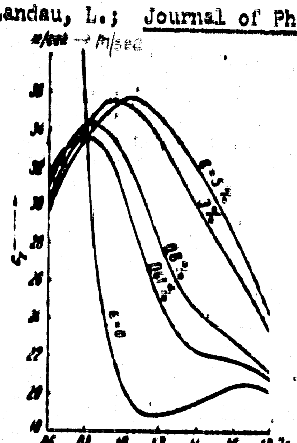

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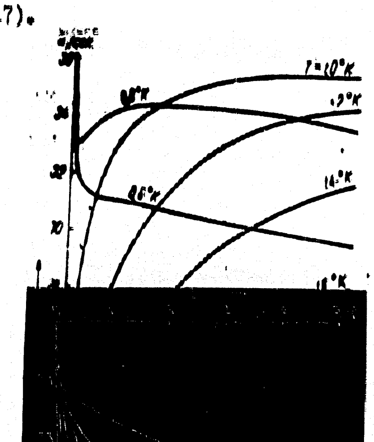

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